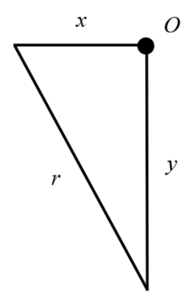
Exercise 17

Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?

Solution

Assume the two cars start at the origin O. The rate that r, the distance between the cars, is changing after two hours is unknown.



The Pythagorean theorem gives the relationship between the sides of the triangle.

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

Differentiate both sides with respect to t.

$$\begin{aligned} \frac{dr}{dt} &= \frac{1}{2}(x^2 + y^2)^{-1/2} \cdot \frac{d}{dt}(x^2 + y^2) \\ &= \frac{1}{2}(x^2 + y^2)^{-1/2} \cdot \left(2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}\right) \\ &= \frac{1}{\sqrt{x^2 + y^2}}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right) \end{aligned}$$

The sides of the triangle after two hours are $x = 25(2) = 50$ mi and $y = 60(2) = 120$ mi. Therefore, the rate that the distance between the cars increases after two hours is

$$\left.\frac{dr}{dt}\right|_{\substack{x=50\\y=120}} = \frac{1}{\sqrt{50^2 + 120^2}}\left[50\left(25\ \frac{\text{mi}}{\text{h}}\right) + 120\left(60\ \frac{\text{mi}}{\text{h}}\right)\right] = 65\ \frac{\text{mi}}{\text{h}}.$$